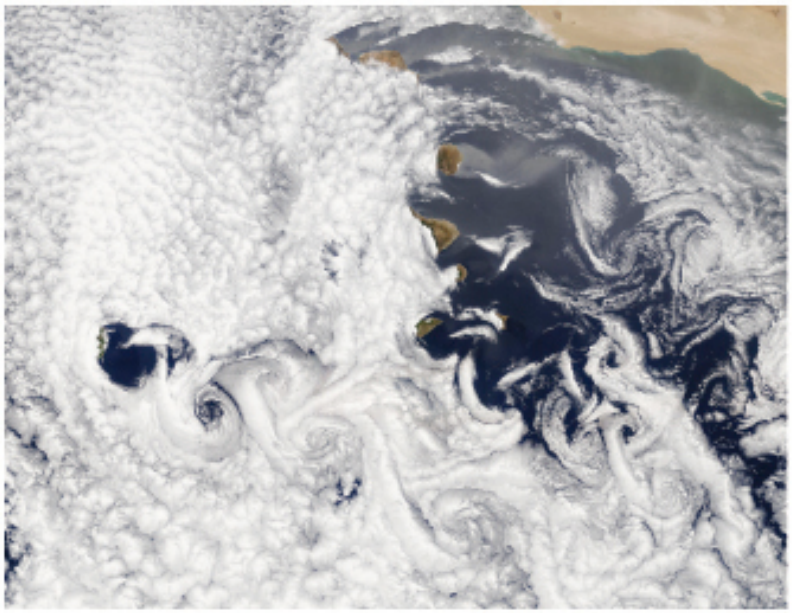
Ocean Modeling - EAS 8803

Barotropic Instability

- A simple example
- What makes a wave grow unstable
- When is the flow unstable
- What is the growth rate of instability
- Revisiting the simple example



Introduction to Geophysical Fluid Dynamics

Physical and Numerical Aspects

Benoit Cushman-Roisin and Jean-Marie Beckers

Academic Press

Chapter 10

Primitive Equations

$$
\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \\
0 &= -\frac{\partial p}{\partial z} \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0,
\end{aligned}
$$

An example of Barotropic Instability

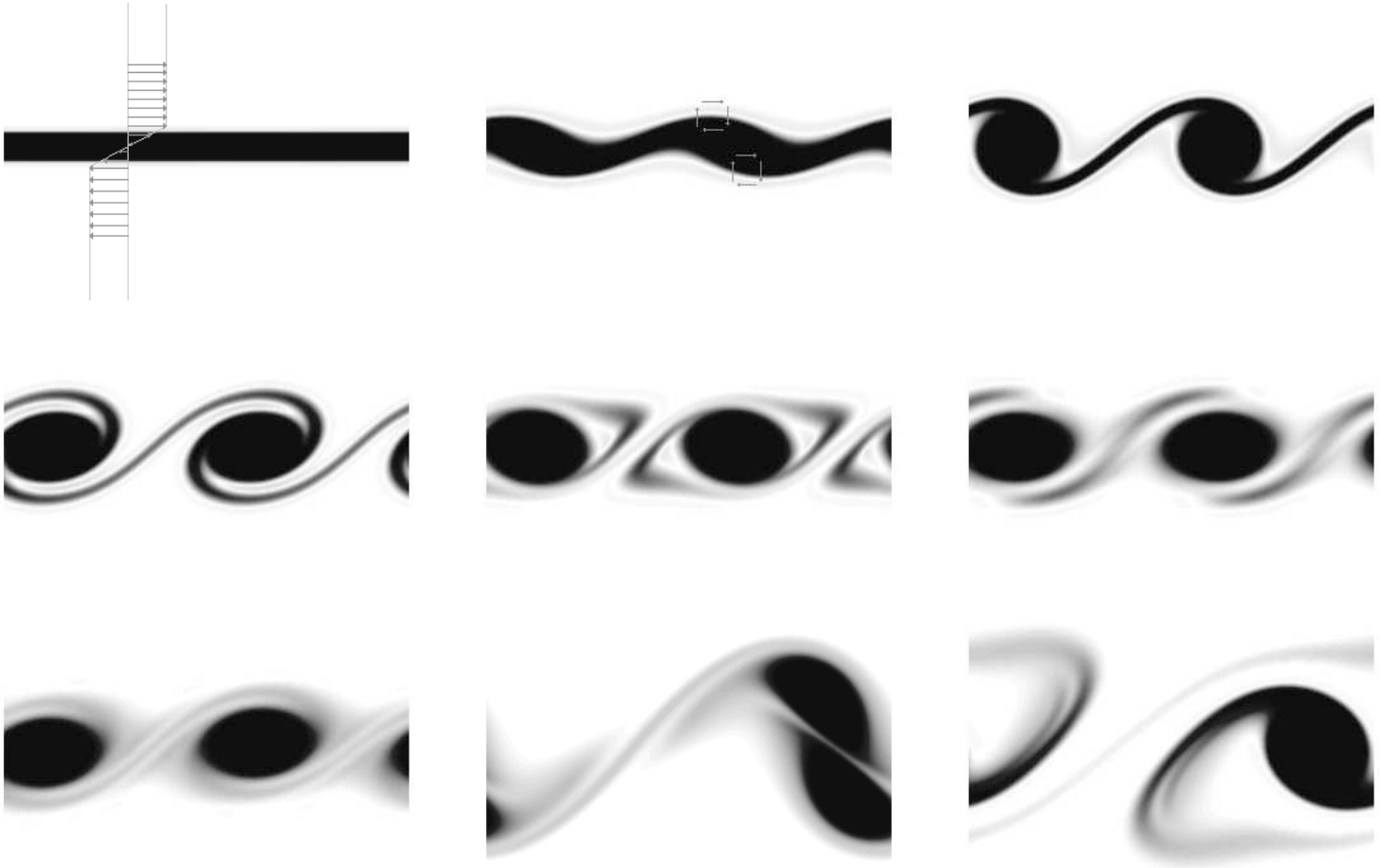


Figure 10-4 Finite-amplitude development of the instability of the shear flow depicted in Figure 10-2. The troughs and crests of the wave induce a vortex field, which, in turn, amplifies those troughs and crests. The wave does not travel but amplifies with time. [The sequence of figures shown here were generated with `shearedflow.m` developed in Chapter 16].

An example of Barotropic Instability

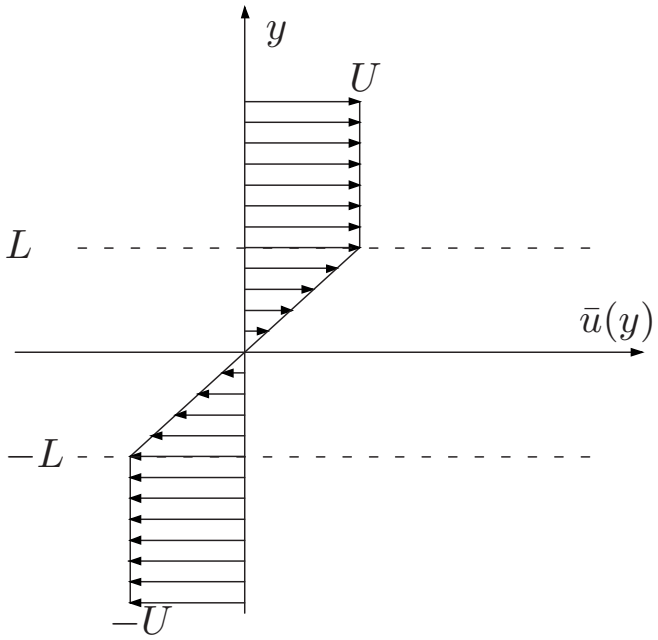
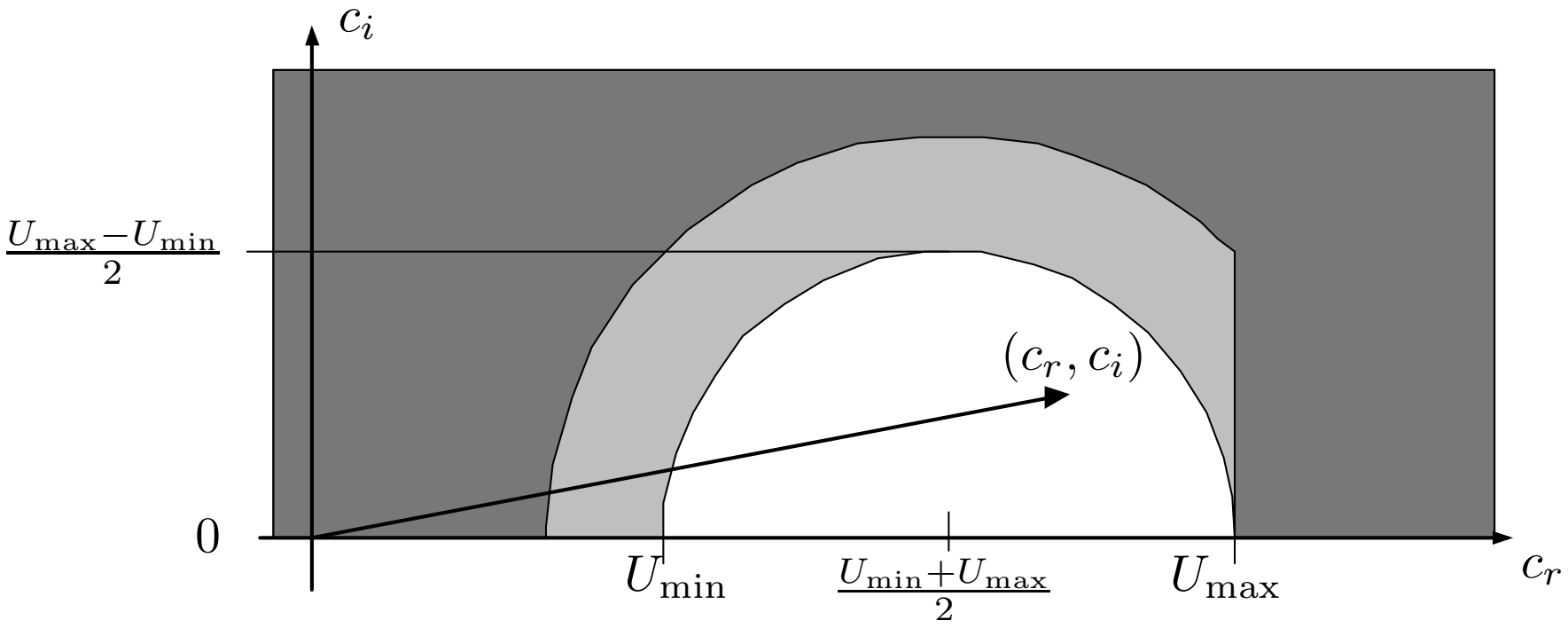


Figure 10-2 An idealized shear-flow profile that lends itself to analytic treatment. This profile meets both necessary conditions for instability and is found to be unstable to long waves.



$$U_{\min} - \frac{\beta_0 L^2}{2(\pi^2 + k^2 L^2)} < c_r < U_{\max}$$