ADVANCED ENVIRONMENTAL DATA ANALYSIS

Special Homework on Time Processes

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1) A simple forced system

Consider the following dynamical system:

$$\frac{dy(t)}{dt} = f(t) - \varepsilon y(t)$$

where $f(t)$ is a given forcing and $\varepsilon = 1/10\Delta t$ is the rate of decay or damping.

Question 1 – Numerical Simulation

Solve this equation numerically on a computer using an Euler Scheme for the two cases below between the time interval $t = \left[0..660\Delta t\right]$ and with initial condition $y(t=0)=0$

Case 1) $f(t) = n(t)$ white noise (e.g. stochastic forcing) with gaussian distribution and standard deviation = 1.

Case 2) $f(t) = n(t) + 0.5\sin\left(\frac{2\pi}{36\Delta t}t\right)$ is white noise with the addition of a sine wave with frequency $\omega = 2\pi / 36\Delta t$.

Plot the numerical solutions and describe the differences very briefly.

Question 2 – Auto-correlation Function

Now compute and plot the auto-correlation function $\alpha(\tau)$ for both cases and comment on the differences. What can you infer from inspection of $\alpha(\tau)$?

Question 3 – Power Spectrum

Decompose the output timeseries using the Fourier Series and plot the power spectrum using loglog scale for both the spectrum $\hat{y}(\omega)^2$ and the frequency ω. You will find that your plot will look a bit noisy. Simulate the model case 1 and 2 by generating an ensemble of timeseries using a different realizations of noise each time. For each realization plot the spectrum and at the end plot the average spectrum. Comments on the results.

2) A forced and damped oscillation system

Consider the following dynamical system:

$$\frac{d^2 y(t)}{dt^2} + \omega^2 y(t) = f(t) - \varepsilon \frac{dy(t)}{dt}$$

where $f(t)$ is a given forcing, $\varepsilon = 1/20\Delta t$ is the rate of decay or damping and $\omega = 2\pi/25\Delta t$.

Question 1 – Numerical Simulation

Solve this equation numerically on a computer using at least a third order scheme for the 2nd derivative and an Euler Scheme for the first derivative. Consider cases below between the time interval $t = \left[0..660\Delta t\right]$ and with initial conditions $\mathrm{y(0)=cos(}\omega * 0)$ $\mathrm{y(\Delta t)=cos(}\omega * \Delta \mathrm{t)}$ (you need two time levels to initiate the numerical scheme)

Case 1) $f(t) = 0$ and $\varepsilon = 0$

Case 2) $f(t) = 0$

Case 3) $f(t) = n(t)$ white noise (e.g. stochastic forcing) with gaussian distribution and standard deviation = 1.

Case 4) $f(t) = n(t) + 0.5\sin\left(\frac{2\pi}{36\Delta t} t\right)$ is white noise with the addition of a sine wave with frequency $\omega = 2\pi/36\Delta t$.

Plot the numerical solutions and describe the differences very briefly.

Question 2 – Auto-correlation Function

Now compute and plot the auto-correlation function $\alpha(\tau)$ for all cases and comment on the differences. What can you infer from inspection of $\alpha(\tau)$?

Question 3 – Power Spectrum

Decompose the output timeseries using the Fourier Series and plot the power spectrum using loglog scale for both the spectrum $\hat{y}(\omega)^2$ and the frequency ω. Comments on the results.

3) The El Niño climate process

We are now going to diagnose a real climate process called the El Niño Southern Oscillation (ENSO). Begin by downloading and plotting the Niño34 index at monthly resolution found here https://www.esrl.noaa.gov/psd/gcos_wgsp/Timeseries/Data/nino34.long.data

Question 1 - Power Spectrum

Decompose the output timeseries using the Fourier Series and plot the power spectrum. However, instead of plotting the spectrum as a function of frequency, plot it as a function of period and do not use a loglog scale.

You will find that it is hard to identify which peaks are significant in the spectrum. To better diagnose the dominant periodicity in the Niño34 index and their significance, we will use two complementary approaches.

Question 2 – Simulate the process with an Auto-Regressive Model of order N

Even though you do not have the exact equation of the ENSO process, we will try to use an auto-regressive model of order 4 (AR-4) to simulate the Niño34 timeseries.

$$nino(t+1) = a_0 nino(t) + a_1 nino(t-1) + a_2 nino(t-2) + a_3 nino(t-3) + n(t)$$

Using different realizations of noise $n(t)$ simulate 100 synthetic Niño34 timeseries that have the same length and variance of the original data. For each simulated timeseries compute the spectrum and then plot the average spectrum. Is it easier to identify peaks in variance? Try this again but use an AR-8. Does the fit to the spectrum change?

Question 3 – Significance Testing using Red Noise Null Hypothesis

We are now going to test if the peaks in the spectrum are significant with respect to a null hypothesis that assumes an auto-correlated timeseries with no significant periodicity. This null hypothesis is equivalent to an Auto-regressive process or order 1 (AR-1).

Generate 1000 realizations of an AR-1 process and compute for each one the spectrum. Then plot the average spectrum and the 95% tile of the spectrum (e.g. 2 standard deviations). Super-impose the Nino34 observed spectrum and determine which peaks are above the 95% tile.

Note: The AR-1 model has to generate timeseries that have the same variance as the Niño34 index