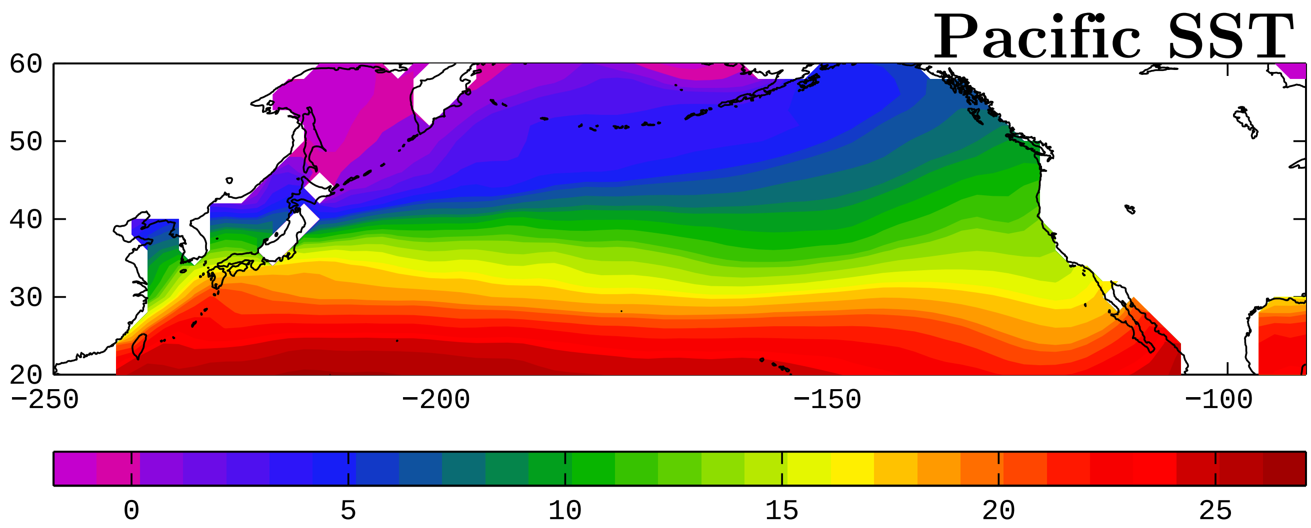
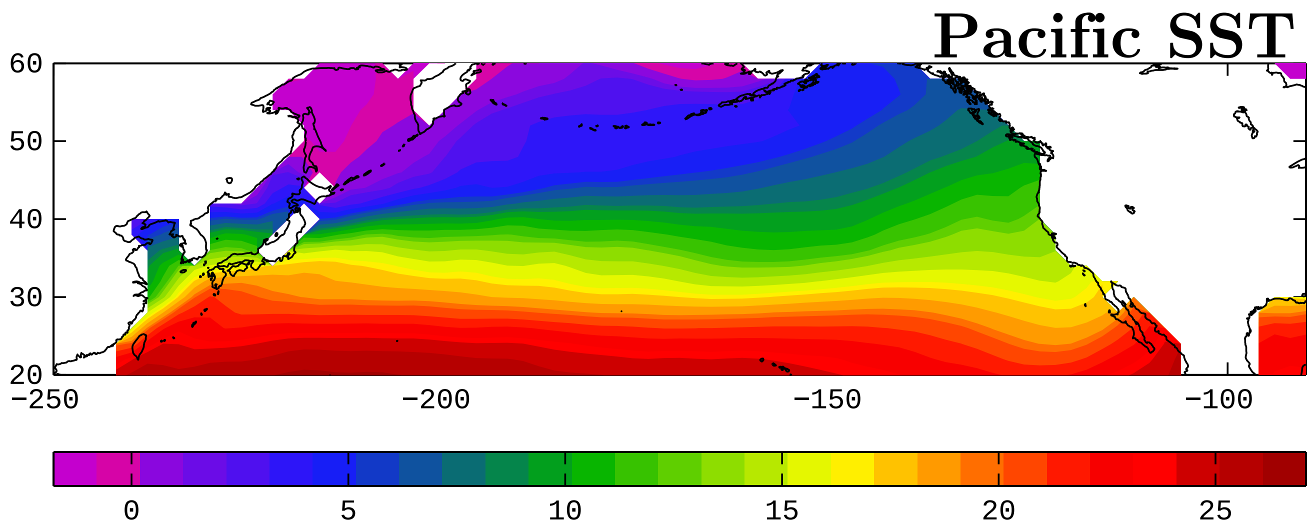
Practical application of least square theory

* Function Fitting
* Inverse problem

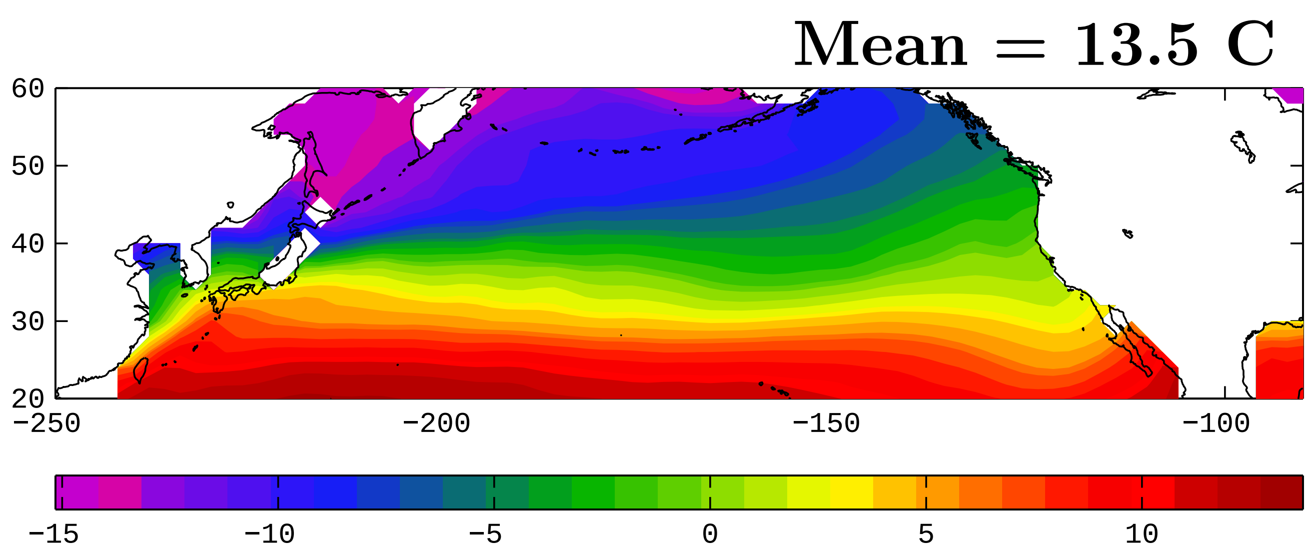
I want to compute SST anomalies



Which mean do I remove?

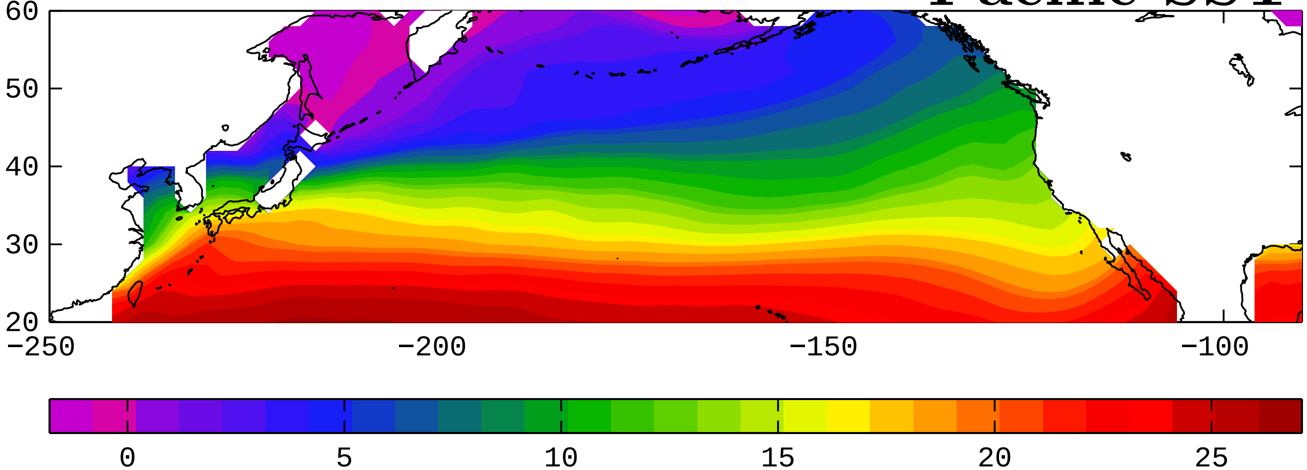


OPTION 1: Remove the constant spatial mean

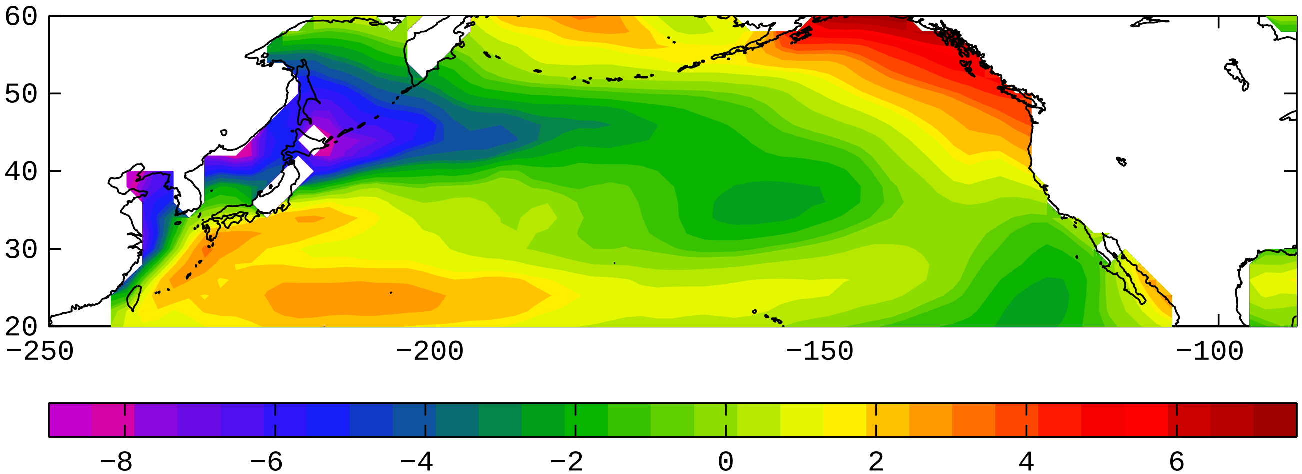


Function Fitting

Pacific SST

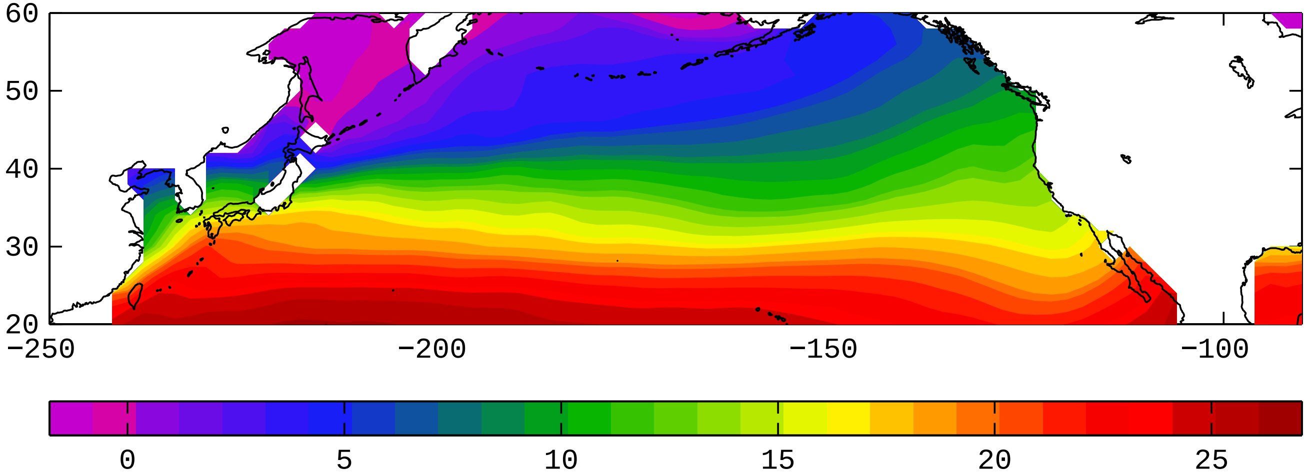


OPTION 2: Remove the meridional gradient (better)

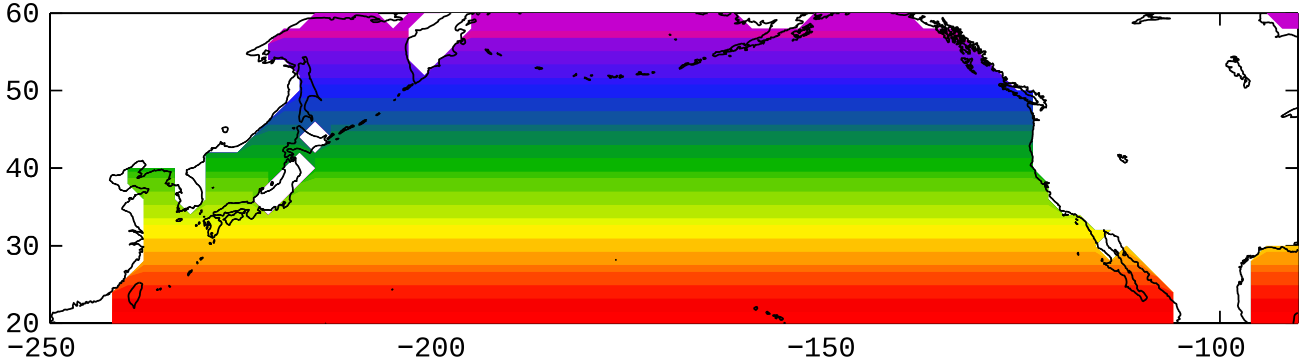


Function Fitting

How to remove the gradient with least square?



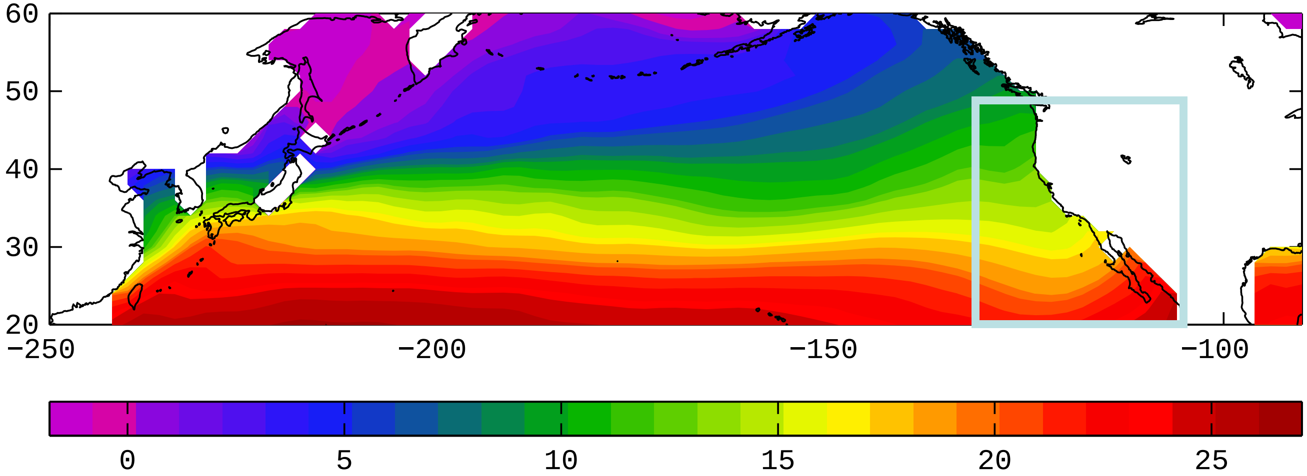
Spatial Mean Gradient removed

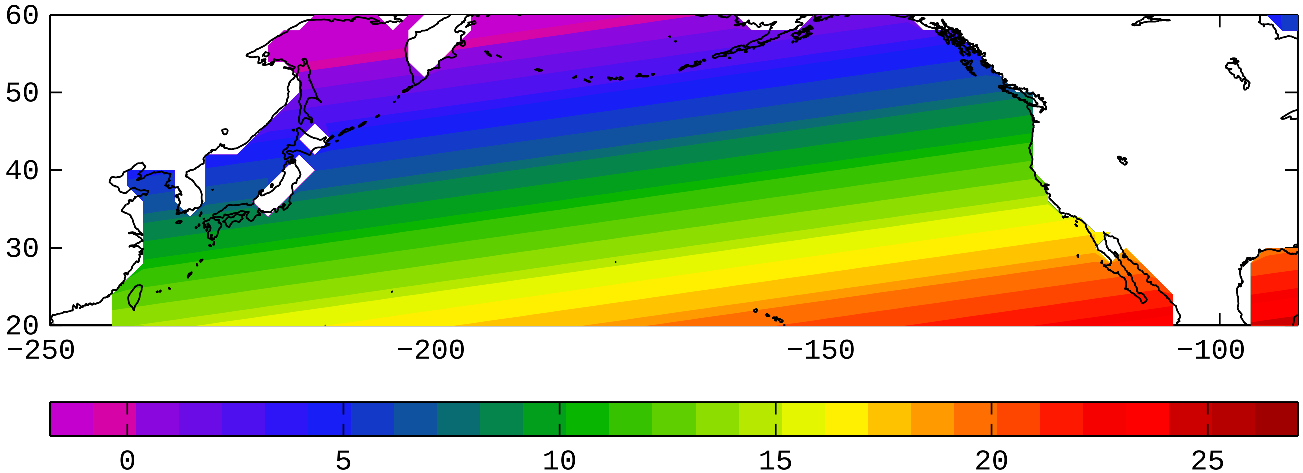


(1) Least Square Solution

$$\begin{cases} J = \left(\mathrm{y} - \mathrm{Ex}\right)^T \left(\mathrm{y} - \mathrm{Ex}\right) \\ \hat{\mathrm{x}} = \left(\mathrm{E}^T \mathrm{E}\right)^{-1} \mathrm{E}^T \mathrm{y} \end{cases}$$

What if we want to remove the gradient associated with the California Current?

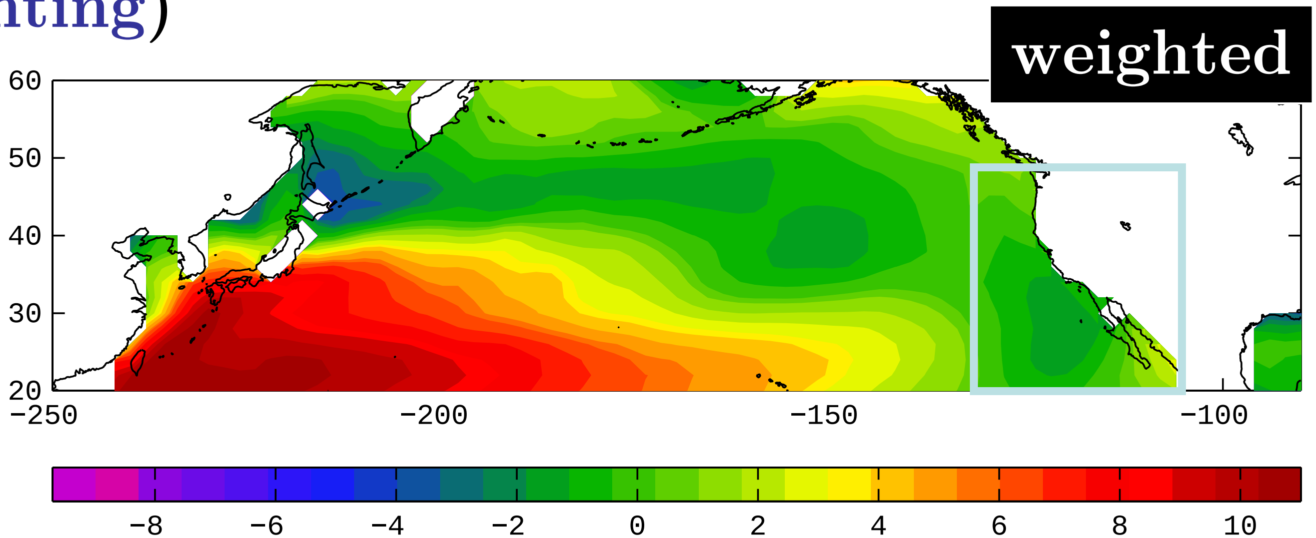


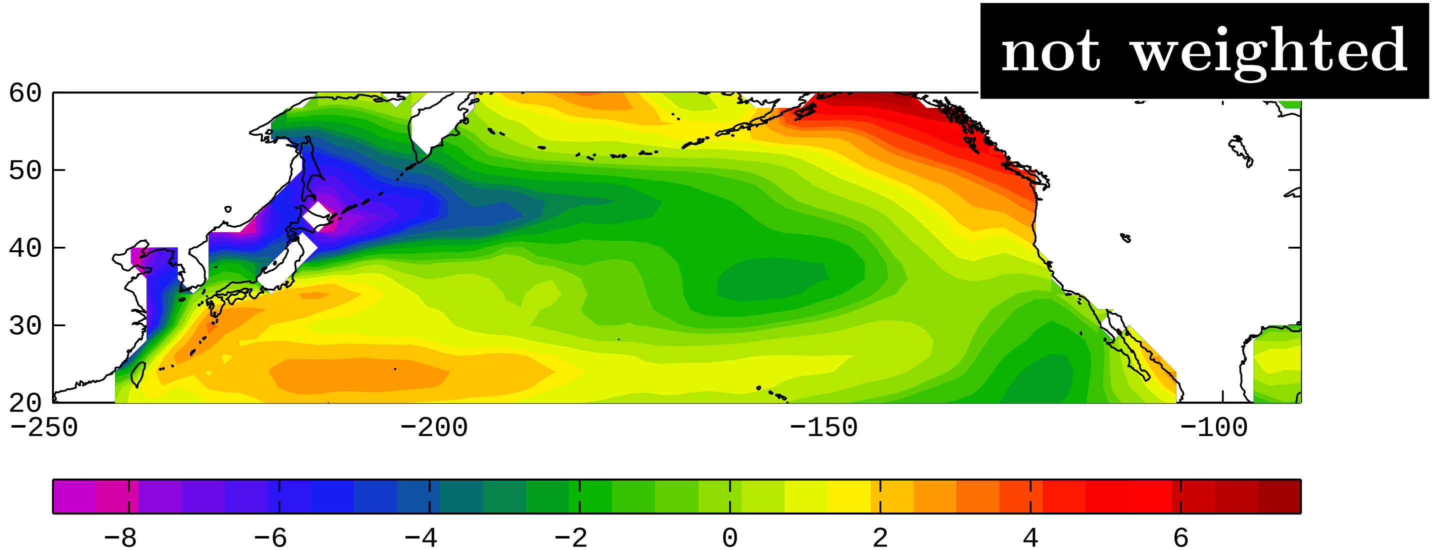


(2) Weighted Least Square Solution

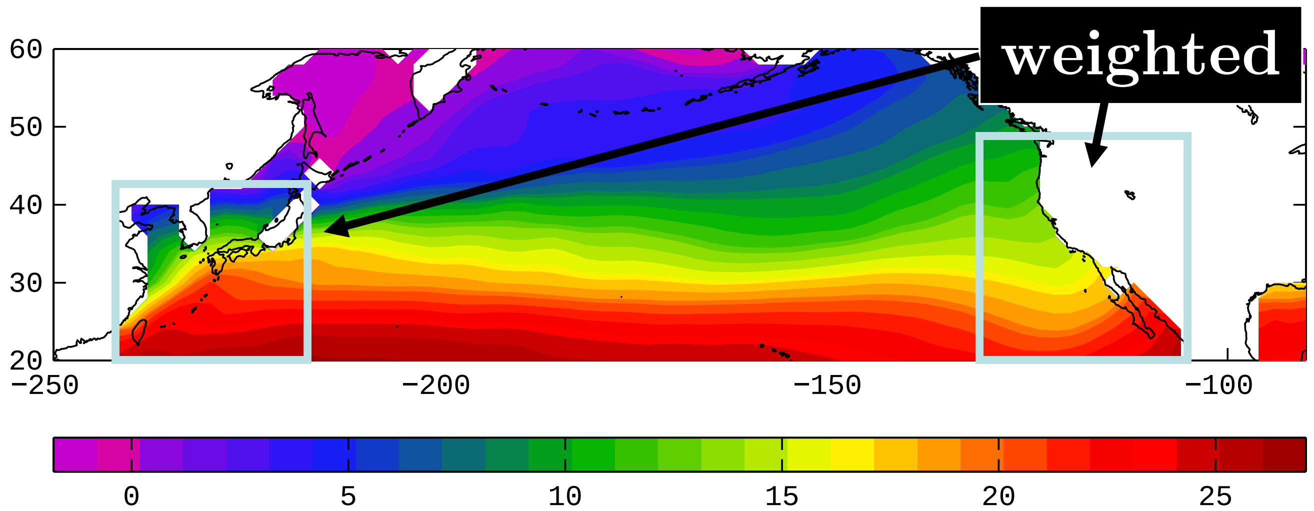
$$
\begin{cases}
J = \left(\mathrm{y} - \mathrm{E}\mathrm{x}\right)^{T} \mathrm{W} \left(\mathrm{y} - \mathrm{E}\mathrm{x}\right) \\
\hat{\mathrm{x}} = \left(\mathrm{E}^{T} \mathrm{W} \mathrm{E}\right)^{-1} \mathrm{E}^{T} \mathrm{W} \mathrm{y}
\end{cases}
$$

Remove the meridional gradient (with different weighting)

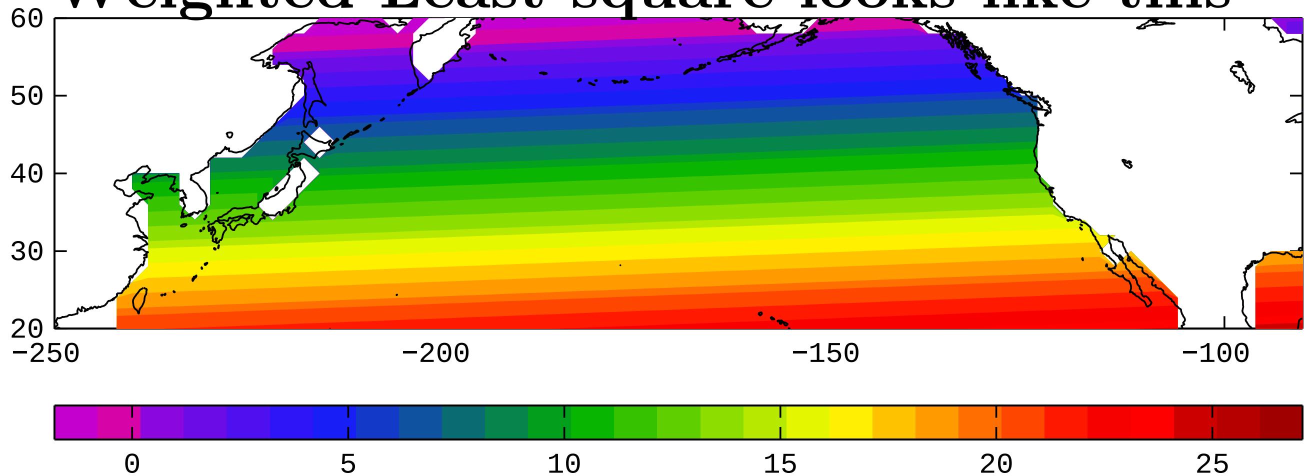




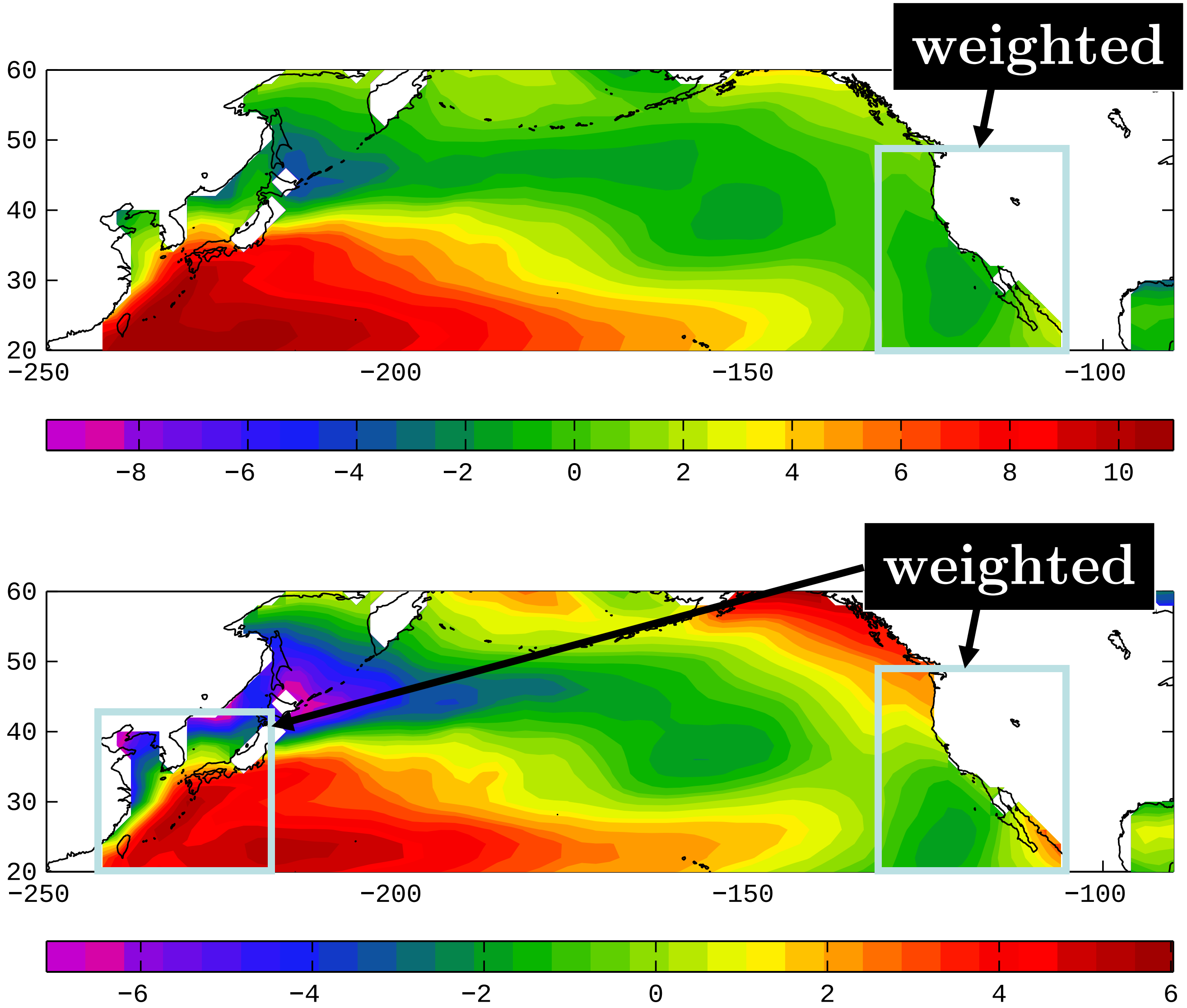
Equally weighted California Current and Cina Sea



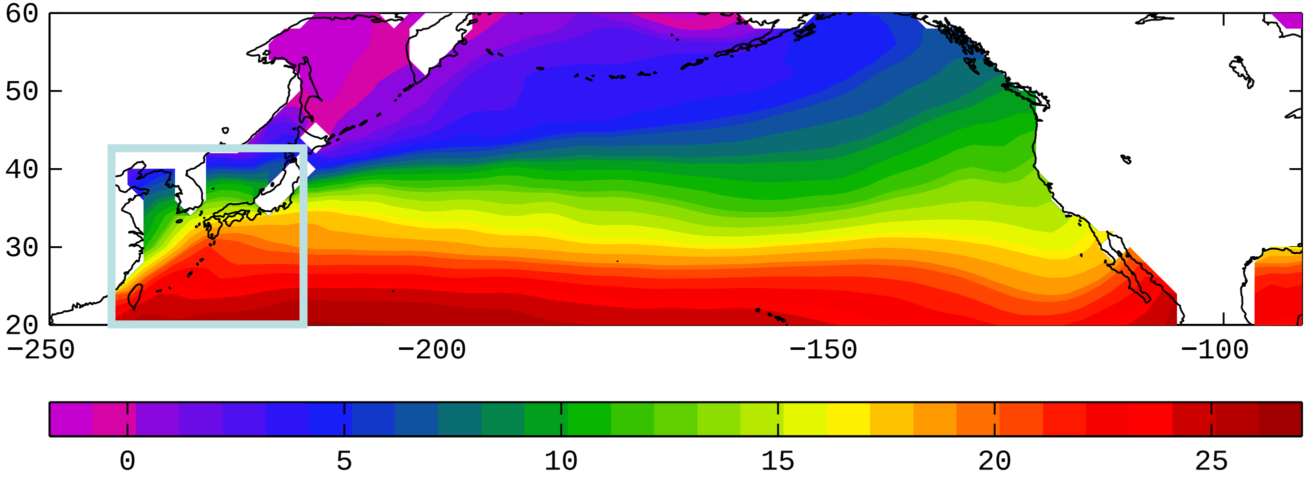
Weighted Least square looks like this



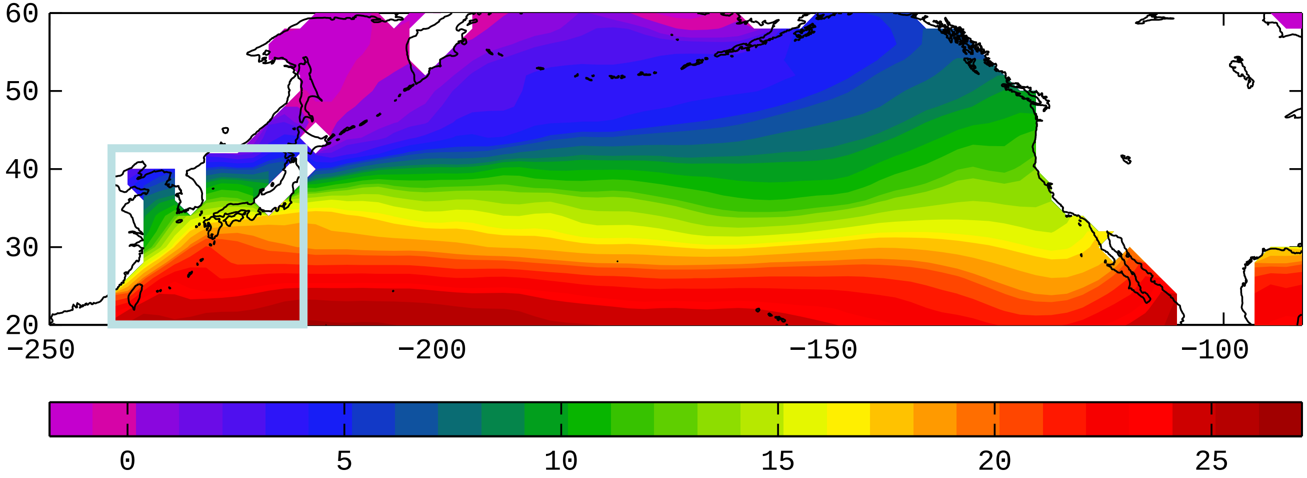
Anomalies (with different weighting)



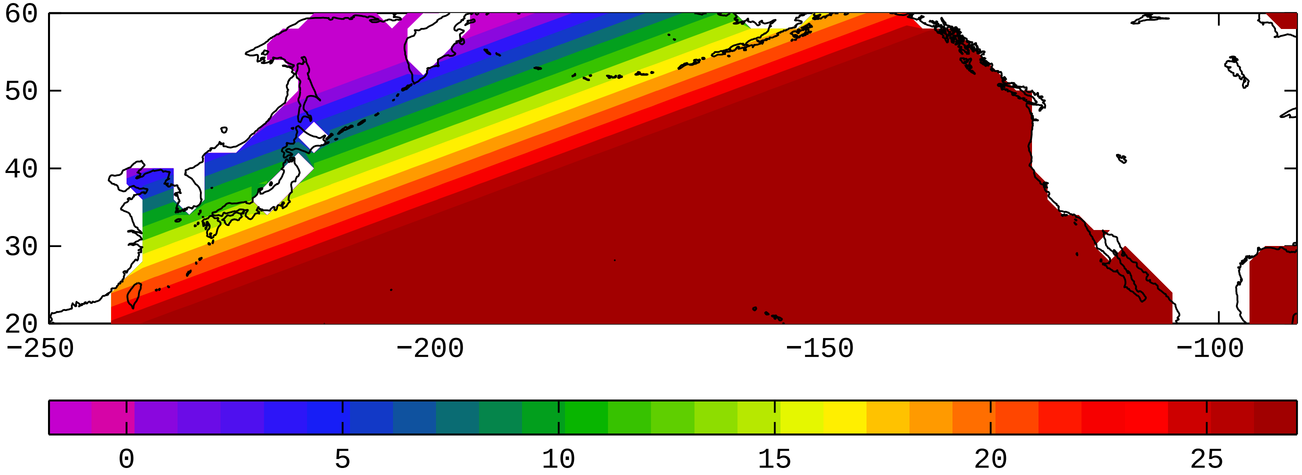
What if we want to remove the gradient associated with the South Cina Sea?



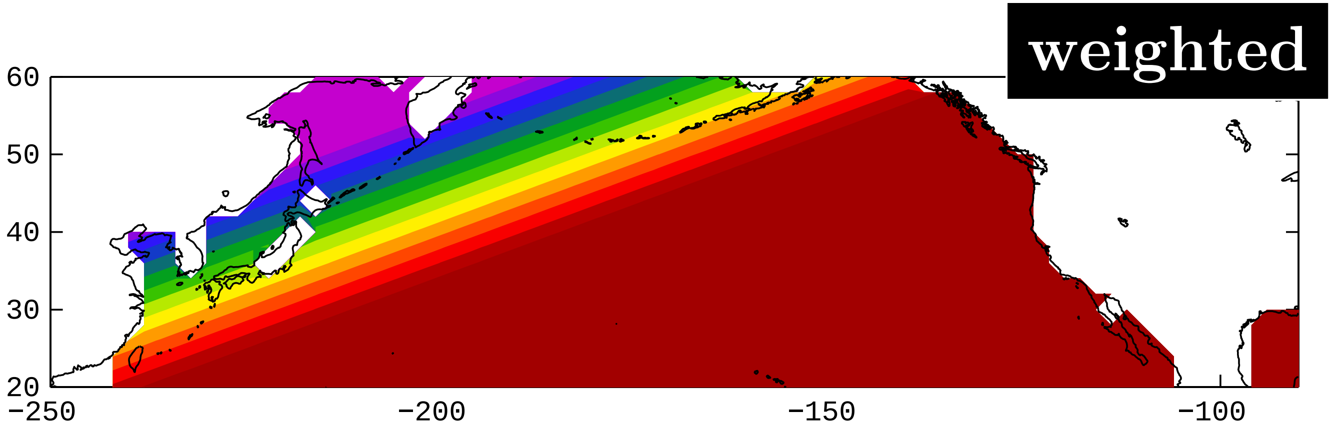
What if we want to remove the gradient associated with the South Cina Sea?



Not so happy!

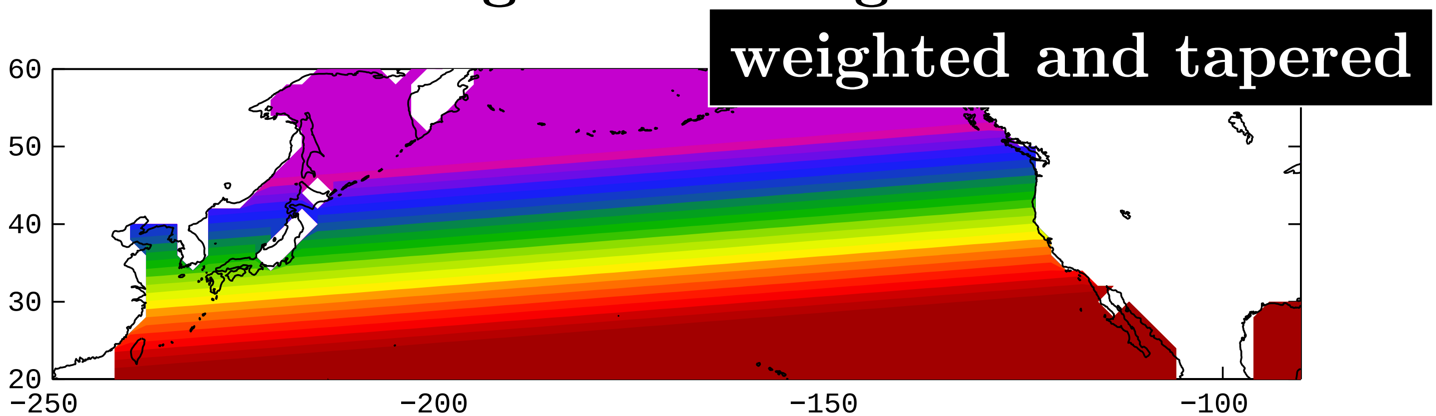


Previous bad solution



Limit the size of the model parameter, in particular The parameter controlling the zonal gradient

New better solution



(3) Weighted and Tapered Least Square Solution

$$\begin{cases} J = \left(y - Ex\right)^T W \left(y - Ex\right) + x^T S x \\ \hat{x} = \left(E^T W E + S\right)^{-1} E^T W y \end{cases}$$