EAS 8803 – Environmental Data Analysis –Exam

(Pages 1/3)

Question 1:

Part 1) The test of variance says the confidence limit is $\frac{\nu}{\chi^2_{97.5\%,d.o.f.}} s^2 < \sigma^2(s) < \frac{\nu}{\chi^2_{2.5\%,d.o.f.}} s^2$

where $\sigma^2 =$ true variance, $s^2 =$ sample variance and ν = degree of freedom in the estimate of s^2

a) Given that the sample variance is computed from a variable $x(t_n)$ with $n = 1...N$ derive the identity $\nu s^2 = \sigma^2 \chi^2$ where ν are the degrees of freedom in computing s^2

b) Suppose $x(t_n)$ has an autocorrelation $\alpha(\Delta T) = e^{-\Delta T / 4\Delta T}$ and length N. How does that affect the confidence limit? Will they be higher or smaller?

Part 2) Now assume that you have two series y(t) and x(t) with normal stationary statistics and length *N*. You computed the correlation coefficient and you want to establish if the correlation is significant.

a) Using the concepts learned above (namely how to asses the confidence limit on variance) explain how you would go about assessing if the correlation coefficient is significant to the 95% level. (NOTE: you may want to think about the PDF of a new variable defined as yx)

b) Assume that the PDF of yx has a Gaussian shape. How would the shape of the Gaussian PDF change if the length of the series y(t) and x(t) was *2N*. Make a drawing/sketch.

Question 2:

Part 1) Given a series y(t) with stationary statistics.

a) Define what it means to be stationary.

b) Derive the autocovariance function $\alpha(\tau)$ where τ is the time lag.

c) Show that the Fourier Transform of $\alpha(\tau)$ is equal to the spectrum $\hat{y}(s)\hat{y}^*(s)$

Part 2) Assume now that y(t) is a continuous function given by the following Fourier Series $y(t) = \sum_{n=1}^{N}\left[A_{y,n}\cos(2\pi s_n t) + B_{y,n}\sin(2\pi s_n t)\right]$ and $x(t) = A_{x,10}\cos(s_{10}2\pi t + \theta)$.

a) What is the covariance of y(t) with x(t)? (You can just write it down).

b) Are the signals in phase at the frequency s_{10}? If not what is there phase difference?

Part 3) Now let us be more general and assume that you have two series y(t) and x(t) with stationary statistics.

a) Derive the covariance function $\alpha(\tau)$ between the two series.

b) Show that the Fourier Transform of $\alpha(\tau)$ is equal to the spectrum $\hat{y}(s)\hat{x}^*(s)$. Is $\hat{y}(s)\hat{x}^*(s)$ going to be real or complex?

c) Based on what you learned about spectra, can you give an interpretation of the meaning of $\hat{y}(s)\hat{x}^*(s)$

Question 3:

Part 1) Assume you have the following dynamics $\frac{d^2y}{dt^2} = -s_b y + f(t) + A\cos(2\pi s_a t)$ and you computed the spectrum of y(t). You see some peaks and you want to know if they are significant.

a) Without solving for the spectra how many significant peaks would you guess and at what frequency? Make a sketch.

b) Draw the autocorrelation function and spectrum of the following processes: white noise, red noise, y = at, $y = a\cos(2\pi S_a t)$

Question 4:

Assume $\underline{y} = \underline{\underline{A}}\,\underline{m} + \underline{n}$ where $< \underline{n}\,\underline{n}^T > = \sigma_n^2 \underline{\underline{I}}$. $\underline{\underline{A}}$ is a known MxN matrix and $\underline{y}$ a measured quantity.

a) When is the system $\underline{y} = \underline{\underline{A}}\,\underline{m} + \underline{n}$ overdetermined and underdetermined?

b) Find $\underline{\hat{m}}$ using LSQ?

c) Now assume I told you $< \underline{m}\,\underline{m}^T > = \underline{\underline{P}} = \begin{bmatrix} \sigma_m^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_m^2 \end{bmatrix}$. Find $\underline{\hat{m}}$ using this information.

d) Estimate the uncertainty on your estimate of $\underline{\hat{m}}$ by computing $\hat{\underline{\underline{P}}} = < (\underline{m} - \underline{\hat{m}})(\underline{m} - \underline{\hat{m}})^T >$.