Covariance Modeling in the SPACE DOMAIN and Multivariate optimal interpolation

Assume you have

$sst(lon, lat, time)$ → sea surface temperature

$p(lon, lat, time)$ → precipitation data

Goal: use precipitation data to map/reconstruct SSTs.

$$\underline{sst} = \underline{\underline{\alpha}}\,\underline{p} + \underline{n} \qquad \underline{sst} = \begin{bmatrix} sst(lon1, lat1) \\ \vdots \\ sst(lonN, latN) \end{bmatrix}$$

$$\underline{\underline{\alpha}} = \underbrace{\langle \underline{sst}\,\underline{p}^T \rangle}_{\underline{\underline{C}}_{sst,p}} \underbrace{\langle \underline{p}\,\underline{p}^T \rangle^{-1}}_{\underline{\underline{C}}_{pp}^{-1}}$$

$$\langle \underline{n}\,\underline{n}^T \rangle = \underline{\underline{C}}_{sst\,stt} - \underline{\underline{C}}_{sst\,p}\,\underline{\underline{C}}_{pp}^{-1}\,\underline{\underline{C}}_{sst\,p}^{T}$$

"The FIT" ——

After computing the covariance we estimate the sst for 1996 (Fig. 1; 1996_error.pdf) along with the errormap $diag(\langle \underline{n}\,\underline{n}^T \rangle)$.

Fig 1: red * are the locations of precipitation data used for reconstruction. The plot shows the TRUE SST, the reconstructed $\hat{\underline{sst}} = \underline{\underline{\alpha}}\,\underline{p}$, the true error $[\underline{sst} - \hat{\underline{sst}}]$, and the estimated error $\sqrt{diag(\langle \underline{n}\,\underline{n}^T \rangle)}$

One can repeat the fit for all years 1978–2000 and compute the following quantities (Fig2inetn_rec.pdf)

$$\frac{\text{rms}\,(\underline{sst} - \underline{\hat{sst}})}{\text{rms}\,(\underline{sst})} \quad \text{and} \quad \frac{\sqrt{\text{diag}(\langle \underline{n}\,\underline{n}^T \rangle)}}{\text{rms}\,(\underline{sst})}$$

these two quantities should by similar if the model $\underline{\underline{\alpha}}$ is robust and statistic are stationary.

NOTE: Some times the covariances $\underline{\underline{C}}_{yx}$, $\underline{\underline{C}}_{yy}$, $\underline{\underline{C}}_{xx}$ may contain noise, or in other word eigenvectors with very small eigenvalues. It is therefore convinient to filter out the noise by decomposing the covariances in eigenvectors and keeping only the ones with high eigenvalue.

"Assessing ERROR Propagation"

How do errors in $\underline{P}$ propagate in $\underline{\hat{sst}}$?

$$\underline{\hat{sst}} + \underline{\varepsilon}_{sst} = \underline{\underline{\alpha}} (\underline{P} + \underline{\varepsilon}_P)$$

$$\boxed{\underline{\varepsilon}_{sst} = \underline{\underline{\alpha}} \, \underline{\varepsilon}_P}$$

For example if $\underline{P}$ has a signal to noise ratio $SN = 10$

$$\underline{\varepsilon}_P = \frac{\mathrm{diag}(\langle \underline{P} \, \underline{P}^T \rangle)}{SN} = \frac{\mathrm{diag}(\underline{\underline{C}}_{PP})}{SN}$$

An example of the calculation of $\mathrm{abs}(\underline{\varepsilon}_{sst})$ is shown in (Fig. 3, 4 ; err_prop_north.pdf
err_prop_south.pdf).

The figure show the error propagation from the precipitation network (red asterisk) for different $SN = 10, 5$ and 2

"Network Design and Adjoint"

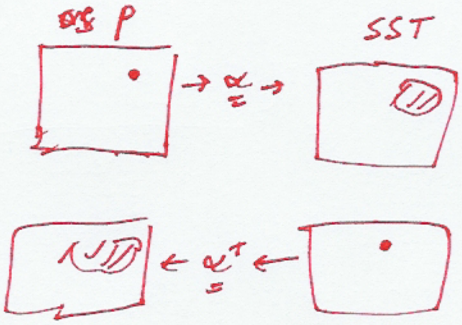
To answer the following question:

"which precipitation stations are most usefull to reconstruct the tropical pacific sst?"

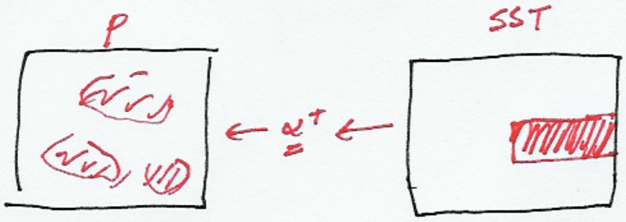
Recall:

$\hat{\underline{sst}} = \underline{\underline{\alpha}}\,\underline{P}$ $\quad \underline{\underline{\alpha}} = \underline{\underline{C}}_{sst\,p}\,\underline{\underline{C}}_{pp}^{-1}$ "forward model"

$\underline{\underline{\alpha}}^T = \underline{\underline{C}}_{pp}^{-1}\,\underline{\underline{C}}_{p\,sst}$ "adjoint model"



The Adjoint model $\underline{\underline{\alpha}}^T = \underline{\underline{C}}_{pp}^{-1}\,\underline{\underline{C}}_{p\,sst}$ gives information of how any given sst location covaries with all other precip. location. We can use this to track where information of the tropical pacific SST is coming from



To do so define a new vector $\underline{sst\_sens} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Ones at the location of tropical pacific

$$\underline{p\_sens} = \underset{=}{\alpha}^T \, sst\underline{\_sens}$$

An example of this calculation done for the TROPICAL PACIFIC, WARM POOL and INDIAN OCEAN is shown in (Fig. 5; netw-design.pdf)

Fig. 5 The red box labels the region of interest e.g. TROPICAL PACIFIC where sst_sens = 1

The red asterisk are the precipitation network used previously to show how well they capture the SST variability in the various regions.