ENVIRONMENTAL DATA ANALYSIS EAS 8803
HOMEWORK #3

Generate on the computer the following timeseries:

$$y(t_n) = 10\sin(w_1 t_n) + 5\cos(w_2 t_n) + 2\cos(w_3 t_n) + n(t_n)$$

where $N = 1000;\ \Delta t = \frac{1}{12}$ years; $w_1 = \frac{2\pi}{N\Delta t}30;\ w_2 = \frac{2\pi}{N\Delta t}50;\ w_3 = \frac{2\pi}{N\Delta t}10;$ and $n(t_n)$ is "white noise" with unit variance and zero mean. Generate the white noise using the MATLAB command `n=randn(N,1);`

a) Compute the power spectra using the FFT. Plot the spectra in a loglog plot and make sure that the area under the curve is equivalent to the variance associated with that frequency. Make the same plot but instead of using frequency on the x-axis use the period. (read Hartmann Chap. 6b page 137-138). See what happens of you do not remove the sample mean of $y(t_n)$ before computing the FFT.

b) Write out the discrete version of the Parseval Theorem and verify that the variance of $y(t_n)$ in physical space corresponds almost exactly to the one computed in frequency space from the Fourier coefficient obtained by the FFT.

c) Repeat (a) and (b) using least squares (LSQ) to fit the coefficients of the Fourier Series.

d) Given the following convolution integral $h(t) = \int_{-\infty}^{-\infty} f(t')g(t-t')dt'$, show that the Fourier transform $\hat{h}(s) = \hat{f}(s)\hat{g}(s)$. (NOTE: the convolution integrals like $h(t) = \int_{-\infty}^{-\infty} f(t')g(t-t')dt'$ appears often. For example when you compute a running average.)