Advanced Data Analysis Homework #4

Due: October 21, 2008

Q1. Generate the following time series (1000 time steps long):

$$y(t) = 2 + 3t + 10\sin(\omega_1 t) + 5\cos(\omega_2 t) + 2\cos(\omega_3 t) + n(t) \quad (1)$$

where $\Delta t = 1$ month, $N = 1000$ total time steps, $\omega_1 = \frac{60\pi}{N\Delta t}$, $\omega_2 = \frac{100\pi}{N\Delta t}$, $\omega_3 = \frac{20\pi}{N\Delta t}$, and $n(t)$ is white noise with unit variance and zero mean. (*Hint:* Use randn in MATLAB to generate white noise).

Compute the power spectrum of (1) for two cases using the Fast Fourier Transform (FFT) routine in MATLAB or IDL: (a) detrending the time series; (b) not detrending the time series. Be sure to read the help of the FFT if you are unsure of the outputs or inputs for the function. Plot the amplitude of the spectrum versus frequency using one of two conventions for plotting the amplitude: (i) you can normalize so that the area under the curve is unity, or (ii) so that the area under the curve matches the total variance in the original time series. Make another plot, but instead of frequency put period on the x-axis. *Be sure to include a plot of the raw data series.*

Discuss the difference that you see between the spectra in (a) and (b) and how it is important in examining and interpreting geophysical time series.

Q2. Read Sections 6.1 and 6.2 of Hartmann's notes on Time Series Analysis and Power Spectra: `http://www.atmos.washington.edu/~dennis/552_Notes_6a.pdf` and `http://www.atmos.washington.edu/~dennis/552_Notes_6b.pdf`. This is a great guide to accompany our lectures and also gives practical guidelines for plotting power spectra and assessing significance.

Download the following time series from the website: HW4_Timeseries1.txt. Calculate and plot the power spectrum for the time series using the analysis constraints outlined below. In all plots, superimpose the power spectrum the theoretical red-noise power spectrum and the 95% significance curve. Discuss the results for each case, and then all cases as a whole. What are the dominant frequencies in this time series?

Here are some useful hints for you:

- You may plot the amplitude in either way as in Q1. Be sure to label the axis appropriately. The x-axis is also your call, but be consistent among all cases.

- To assess degrees of freedom (ν), use Hartmann's estimate: $\nu = \dfrac{N}{M*}$. See the notes for an explanation of the terms (we will ignore f_ω in this homework). Assume $\nu = \infty$ for the red noise fit.

- Remember that $\omega = \dfrac{2\pi k}{N}$, where k is the wavenumber.

- To find the 95% significance curve, multiply the red noise power spectrum by $F_{0.95}$, where F corresponds to the F statistic (search online for tables or look at Hartmann's notes for it).

- The equation for the theoretical red noise power spectrum is also given in Hartmann (Chapter 6, page 160). You will need to calculate the lag-1 correlation of the time series for ρ.

Case a) Lowest resolved frequency: 2π per N. No window tapering and no smoothing.

Case b) Lowest resolved frequency: 2π per N. No window tapering. Smooth the spectra using a 5-point running mean. (e.g., the value of $\phi(3)$ is the average of all the values of the spectrum between $\phi(1)$ and $\phi(5)$).

Case c) Lowest resolved frequency: 2π per $N/10$. No window. No smoothing. (You will average the spectra calculated for 10 subsets of the data, corresponding to 1:100, 101:200, etc.).

Case d) Lowest resolved frequency: 2π per $N/5$. Hanning window. No smoothing. Use an overlap corresponding to half the length of each subset of the data (i.e., 1:200, 101:300,...). Note that since there is an overlap between successive subsets, you will actually be calculating spectra for 9 subsets of the data.

Q3. Download the ENSO index from the website. Using whatever constraints you deem necessary (smoothing, windows, etc.), determine the dominate timescale(s) of variability in the ENSO index. Explain your methodology and how you assess the significance of the spectral peaks. Compare your results with published studies on ENSO frequency (a minimum of 2 references is required).